HYPERGRAVITY AND CATEGORICAL FEYNMANOLOGY

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Abstract: We propose a new line of attack to create a finite quantum theory which includes general relativity and (possibly) the standard model in its low energy limit. The theory would emerge naturally from the categorical approach. The traces of morphisms from the category of representations we use to construct the state sum also admit interpretation as Feynman diagrams, so the non-categorically minded physicist may think of the models as combinatorial expressions in Feynman integrals, reflecting the topology of the triangulated manifold. The Feynman picture of the vacuum would appear as a low energy limit of the theory.

The fundamental dynamics of the theory are determined by a Topological Quantum Field Theory expanded around a conjectured geometric quasi-vacuum.

Although the model is a 4 dimensional state sum, it would have a stringlike ten dimensional perturbation theory.

The motivation for the name "hypergravity" is the existence of an infinite tower of alternating fermionic and bosonic partners of the gravitational field in the theory, with an n=2 chiral supersymmetry functor connecting them. It is remarkable that a supersymmetry appears in the high energy sector of a theory not founded on supergeometry.

1. Introduction

The purpose of this paper is to begin a new attempt to produce a quantum theory of nature, including both general relativity and particle physics, out of the categorical approach. [CKY1,BC1,BC2].

The categorical approach is a method to produce a type of discrete theory, rather like a lattice QFT, but put on a triangulated manifold, instead of a lattice, in order to make it applicable to curved spacetime.

One replaces the fields and Lagrangian of a QFT by an abstract algebraic structure and its operations. A basis for the algebraic structure is used to label sites on the triangulation, and the operations are used to combine these to make numbers called "local contributions".These are then combined into a state sum, which can be viewed as closely analogous to a path integral. The analogy is that the elements of the structure are like local fields, while the structure itself is a sort of Lagrangian density.

In higher dimension than 2, the algebraic structure is often some sort of tensor category and the structure which is used to create the QFT is its associator or something similar.

The local contributions from the relevant tensor category are in fact the evaluations of Feynman diagrams for a special QFT on hyperbolic space[F-K,BC2]. As we shall argue below, this suggests that the Feynmanology of

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the matter fields in the model we are proposing may appear directly as traces of morphisms in the category of representations from which we construct the theory.

The current paper is not self contained. It draws heavily on the categorical state sum models for $B \wedge F$ theory in [CKY1], and the Euclidean and Lorentzian signature state sum models in [BC1,BC2]. We assume that the reader is familiar with those constructions, as well as the fundamentals of category theory. The entire motivation for the proposal is specific features of these models, and of the tensor category from which they are constructed.

The construction of a four dimensional topological state sum (TSS) in [CKY1] assumes a complete tensor category, i.e. one which is closed under the tensor product. In [BC1,2], models were proposed for four dimensional discrete quantum general relativity in Euclidean and Lorentzian signatures respectively. Both models were constrained versions of models of the topological type in [CKY1], which were constructed by restricting to a suitable subcategory which was not a tensor subcategory. The Euclidean model [BC1] was obtained by restricting to the irreducible representations of $U_q so(4)$ which had equal half integer indices for the left and right actions of $U_q so(3)$, which we call "balanced". The Lorentzian model was a similar restriction of the category of representations of the Quantum Lorentz Algebra [BR 1,2], specifically to the irreducible representations of the form $R_{0,p}$. (the general irreducible representation of the QLA is of the form $R_{k,p}$ for k a half integer.) Since these models are not based on closed tensor categories, they are not topological, i.e. they depend on the triangulation used.

In both cases the construction is motivated by the idea that the constraint corresponds to the quantization of the condition that the bivectors on the 2-simplices of a triangulated 4-manifold are simple.

This leaves two difficult problems in the way of developing a serious physical theory:

1. finding the limiting behavior as the triangulation is refined, i.e. the continuum limit

and

2. adding the matter fields.

The behavior of the topological model as the triangulation is refined is not problematical at all. The physical quantities are triangulation independent, so we are already at the continuum limit. The price we pay for this is that there are no local degrees of freedom, not even geometrical ones. The categorical constraint has the effect of breaking topological invariance, so the "smaller" state sum has a "larger" state space.

Several authors have tried to make a continuum limit for the models of [BC1,2] or closely related ones by summing over triangulations or slightly more general combinatorial situations [R-R1,Ba]. The convergence problem for such an approach seems formidable, however.

Our hope is that the two problems have a single solution.

What we want to propose is that the fundamental dynamics of our universe is actually determined by a categorical topological state sum, but that we are in a sort of "bubble", i.e. a region in which labels in the sum are restricted to a subspace of the full labelling set of objects of the category which has a relative stability under the dynamics induced by the TSS. This is similar to the suggestion of Witten [W] that the fundamental theory has a "topological phase" but we fluctuated out of it, except that it is rather a TQFT that has a nontopological quasiphase.

The constrained subtheory would be a summation over some of the labels for the whole theory, but subject to a constraint which has the property that if we begin a state sum with initial conditions satisfying the constraint, the dynamics of the larger TSS would only produce a very small contribution of labels outside the constraint for a long time as measured by a clock constructed within the constrained label space, provided the curvature of the geometry defined by the labelling within the constrained subsystem was small compared to the Planck scale.

Let us formalize matters:

"DEFINITION": A **geometric quasi-vacuum** is a subspace S of the total labelling space T for a TSS such that

1. The constrained labellings can be interpreted as defining a discrete geometry on the underlying manifold.

2. If we calculate the time evolution using the entire TSS, but on initial conditions inside the subspace S, the deviation from the result of the constrained theory will be small provided the curvature of the initial conditions is small in Planck units, for a long time as measured by a clock defined in terms of labels from the set S.

CONJECTURE 1: The Euclidean signature model of [BC1] is a geometric quasi-vacuum for the TSS of [CKY1] with category Rep $U_q so(4)$, i.e. the space of balanced labels is a suitable S in the above definition. The Lorentzian model in [BC2], but with q taken to a root of unity, is a geometrical quasi vacuum of a TSS associated to a version of the QLA with q also a root of unity.

CONJECTURE 2: The set of geometric quasi-vacua associated to the TSSs of conjecture 1 is actually very large, including examples related to string vacua.

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Motivation for these conjectures will appear in the bulk of the paper below.

What we are proposing is that the matter fields we see in the universe are the low energy limit of the other labels in the TSS, besides those that satisfy the constraints for the model for general relativity. If we are in a "bubble", i.e. a region where the labels satisfying the constraints predominate, small fluctuations of the extra labels would appear to propagate through the geometrical background defined by the included labels.

An observer in a region of a TSS which fluctuated into a geometric quasi vacuum would seem to see the world obey the laws of the constrained theory. In particular if the constrained theory approximated the laws of GR coupled to suitable matter, it could easily settle into an expanding solution, so a Planck scale initial fluctuation could appear to become vastly larger to an internal observer. If such an observer attempted to study a region where spacetime was highly curved, then the labels excluded from the constraint would begin to appear, adding at first to the field content of the observed theory. Paradoxically, however, if the observer entered a region which strayed too far from the curvature limitations, and hence from the constrained theory, the additional hierarchy of "fields" would produce a regime in which no local excitations could exist at all anymore, i.e. a topological theory. One way to understand how this would happen is that the new sets of labels which would appear are linked by a supersymmetry functor, so that the corrections which would appear if one refined the triangulation would cancel out in a telescoping supersymmetry.

The way I envision this is that what would appear would be a thermal bath of an ever increasing variety of particles, as predicted by the semiclassical theory in regions of curvature. Eventually,this would reach a critical temperature and make all measurement impossible, destroying any observer in the region.

Put differently, there would not exist a true continuum quantum theory of pure gravity, because at sufficiently high energy densities matter would be formed around the curvature loci. Matter plus gravity would not have a continuum quantum theory, because at still higher temperature the matter fields and metric fields would merge into a topological state sum. Since the higher partners of the labels which give gravity are related by a supersymmetry functor, their contributions to the state sum would telescopically cancel as we refined the triangulation into the region where they became important. Thus, an effect analogous to the no renormalization theorem for supersymmetric theories would cause the state sum to become topological.

This is rather similar to the idea of SUSY breaking, except in a discrete picture, rather than for continuum fields. It is remarkable that the category of unitary representations of the ordinary Lorentz group has a decomposition into partners of the subcategory associated to GR connected by a supersymmetry functor. Infinite supersymmetric hierarchies do not seem to appear in continuum field theories.

The region of spacetime before the fluctuation happened would seem to have

a completely degenerate geometry and a thermal state at the Planck temperature to an observer inside the bubble. This would be a plausible scenario for a big bang.

Any region inside the bubble where the curvature became high enough would also begin to see labels from outside the constraint space, which would appear similar to Hawking radiation, until the Planck temperature was reached.

A region within the bubble which appeared so small that any subdivision of it would be dominated by labellings with large curvature would effectively leave the bubble. An attempt by an observer to probe it would yield a thermal state, and no finer determinateness as to the geometry of the small region. Thus the only continuum limit for the theory would be the topological theory.

The "definition" of a geometric quasi-vacuum given above is motivated by a physical picture, and will need considerable refinement to make it mathematically precise enough for further analysis. We will try to argue that various specific facts about the models of [CKY1 BC1,2] make this picture plausible. **It is with quasi-vacua of the specific TSS related to the unitary representations of the QLA and $U_q so(4)$ that we shall be concerned.** For brevity, and for reasons discussed below we shall refer to them as "hypergravity".

In order to advance the program of this paper, the first necessary step will be to make a precise formulation of the definition of quasi-vacuum in terms of the actual categorical state sum. A natural procedure would be to constrain the labels along a 3D hypersurface in the triangulated 4-manifold to lie within the theory, and to constrain the labels on the 2-simplices incident to the hypersurface both to lie within the constraint subcategory and to correspond geometrically to a small time. One could investigate the behaviour of the state sum for some relatively simple constraint subcategory, then try to prove a quantitative form of the conjecture. Lastly, one could try to generalize the behaviour to a large family of constraint subcategories. We are then conjecturing that a very large family of subspaces of the set of labels would satisfy such a stability condition. If our intuition is correct, we would end up with a picture of the vacuum of the TQFT as a froth of many different types of bubble.

What we want to do in this paper is to make a preliminary analysis of the prospects of this direction of development. There are two points which seem to us to make it worthy of study.

1. There seems to be a natural picture in which the low energy fields produced by the excluded labels would reproduce the standard model. This is because the algebraic structure of the category we are proposing to use makes very natural contact with the ideas of the Connes-Chamseddine model [CC, Coq].

2. A perturbative treatment of the hypergravity model yields a picture of strings with either bosonic or fermionic labels moving through a 10 dimensional

curved space. It is not clear yet if these are related to superstrings as we know them. Nevertheless, this is the motivation for conjecture 2.

In the rest of this paper we will have to treat specific aspects of the CSS's mentioned above, in order to see to what extent the conjectures are justified.

2. Representation theory of the QLA and the Hypergravity Multiplet. Constraints and Quasi-Vacua.

Let us begin by reviewing the basic facts concerning the unitary representations of the classical Lie algebra so(3,1) which is isomorphic to $sl(2,C)_R$. These were first studied by Gelfand, and his collaborator Naimark [GN,N]. There is a fundamental family of these, called the principal series. They can be realized as weighted actions of the group of linear fractional transformations on the Hilbert space $L^2(C)$. They form a two parameter family $R_{k,p}$, where k is a half integer and p is a real number. For any three representations R_{k_i,p_i}, i=1,2,3; there is a unique intertwining operator $R_{k_1,p_1} \otimes R_{k_2,p_2} \to R_{k_3,p_3}$ up to scalar multiple if $k_1 + k_2 + k_3$ is an integer, and there is no intertwiner at all otherwise. The intertwiners are given explicitly in [N].

The constraint for the model of [BC2] is that k=0; i.e. we consider only a state integral over the 10J symbols for the $R_{0,p}$'s.

In order to get a finite model in [BC2], we passed to representations of the quantum group associated to the Lorentz group, the QLA of [P-Wo, Pu, BR 1,2]. The representation theory of the QLA, for a real deformation parameter q is well analyzed in the above papers, and is quite similar to the picture for the ordinary Lorentz algebra, except that the continuous parameter p only takes values in the interval $[0, \frac{4\pi}{ln(q)}]$. The picture for intertwiners is also analogous to the result cited for the Lorentz algebra. The result was that it was possible to write down a state integral model using the representations of the QLA, with all the integrals over finite measures, and hence finite.

Now we would like to think of the constrained model of [BC2] as a submodel of a TSS. This is not quite possible for real values of q, since the sum over the discrete parameter k would still be infinite.

However, it is natural to conjecture that if we passed to q a root of unity, then there would be a truncation of the family of representations $R_{k,p}$ at a sufficiently large k for q a fixed root of unity. The reason to believe that is that the representation $R_{k,p}$ decomposes as an ascending chain of representations of $U_q su(2)$ with the lowest spin in the chain given by k. Thus, since the representations of $U_q su(2)$ with spin greater than (lnq/4i)-1 have quantum dimension 0 and decouple, so should the representations of the QLA at the same k. Assuming this conjecture is correct, and it is certainly not too hard to investigate, We can embed the Lorentzian model of [BC2] as a constrained subsum of a TSS. We already know that the Euclidean signature model of [BC1] can be embedded as a subsum of a TSS, namely the model of [CKY1] with category $RepU_q so(4)$ for q a root of unity.

In either case, it is interesting to think how the rest of the labelling category appears in relationship to the subcategory we are using as a discrete model for gravity. We shall discuss the Lorentzian signature, the Euclidean situation is very similar. The labelling objects appear as a series of copies of the labels for gravity, one partner set $R_{k,p}$ for each half integer k. The couplings for each partner set would be similar to the ones in the gravity labelling set, but different sets would be coupled more weakly to one another by some exponential factor involving the difference in k [BR 2].

Thus we would find a hierarchy of "partners" to gravity, alternately bosonic and fermionic.

There is also a natural operation in the category which connects the adjacent partner sets. Tensoring with a finite dimensional representation (necessarily nonunitary) then projecting back to the unitary category would send each $R_{k,p}$ to a combination of other k's determined by the usual Clebsch Gordon formula, but only with the identical p [BR3]. In particular, tensoring with either the (1/2,0) or (0,1/2) representations (the complexification of the QLA is isomorphic to the product of 2 copies of $U_q sl(2, C)$) would give an operation on the hierarchy of partner sets, mapping each $R_{k,p}$ to $R_{k+1/2,p} \oplus R_{k-1/2,p}$. Since tensor product is functorial, the morphisms of the category, which we are using to construct our diagrams, are also acted on by the supersymmetry functors.

This is the motivation for the name "hypergravity". If we think of the labelling category for the TSS as an extension of the labels for the model for GR, it has the appearance of a hierarchy of copies of the GR labels, alternatively fermionic and bosonic, together with two natural chiral fermionic maps relating adjacent elements of the hierarchy. This is reminiscent of supersymmetric field theories, except that the hierarchy seems to be infinite. The coupling of each "hyperpartner" to itself mirrors the gravity multiplet.

The above cited result about coupling of different partner sets is one motivation for conjecture 1 above. For small curvatures, it would be improbable to see k fluctuate by an entire half unit. Another piece of evidence is the fact that if we restrict the models of [BC1,2] to configurations corresponding to flat metrics, they in fact become topological themselves [CKY1]. Of course the idea that "nearly flat" metrics are "nearly topological"needs a careful quantitative study.

At this point, I hope it is clear that "what would be the low energy effective field content of the hypergravity multiplet?" is a plausible question. One could try to find an analog of the renormalization group for categorical state sums, and try to discover how much of the hypergravity multiplet was relevant.

In the next section we explore the possibility that, because of an algebraic coincidence of noncommutative geometry which does not appear to have a classical geometric analogue, the standard model might well emerge naturally within hypergravity.

3. Connections with the Standard Model and the Connes-Lott

Model

In [C-C], it was argued that the standard model emerges naturally provided an algebra which the authors called the "world algebra" appeared as a symmetry algebra. The algebra they meant was

$$W = C \oplus H \oplus M^3(C) \ .$$

It was incidentally mentioned in their work, and later elaborated in [Coq], that this algebra emerges as the semisimple part of the quantum group $U_q sl(2, C)$ when we let q be a third root of unity.

Note that this is different from the common truncation at a 4rth root of unity. This will be still another new algebraic question to study in order to investigate this model.

Now let us remind ourselves that the QLA is a "real form" of $U_q sl(2, C) \otimes U_q sl(2, C)$, more precisely, that its complexification is exactly that algebra [BR1].

Hence a truncation of the QLA at a third root of unity would contain two chiral copies of the "world algebra"as its semisimple part.

It is not unreasonable to expect that a nilpotent piece of an algebra would vanish in a low energy (=long distance) limit, leaving the world algebra as the effective symmetry of the theory.

This suggests that it might be interesting to examine other odd roots of unity as well, to see if they predict hierarchies of fields which include both the standard model and others which might be more massive, but potentially discoverable.

It is not possible at this point to demonstrate that the standard model actually emerges from the hypergravity multiplet at q a third root of unity. The truncated representation theory needs to be studied, to determine how the representations of our category decompose under the action of the world algebra. In addition, a form of the renormalization group needs to be developed for CSS's, and many hard calculations need to be done. Nevertheless, in QFT symmetry is a very strong principle, and seeing the symmetry of the standard model emerge by accident in the hypergravity picture is very surprizing at least.

4. Categorical Feynmanology

In our proposal, there is no true (continuum) QFT except TQFT. The labels that appear as matter fields in a certain regime merge into a topological sum in the continuum limit. Thus the effective theory of matter should be a combinatorial, algebraic one, which could appear within a tensor category.

Such a point of view already exists in QFT as practiced, namely, that of Feynman.

It has long been one of the disturbing problems of theoretical QFT that neither the derivation of the Feynman rules from a QFT nor the reconstruction of a QFT from Feynman amplitudes is mathematically sound. On the other hand, it is rather the Feynman rules than the continuum QFT which have directly been verified by experiment.

All this makes us wonder if the Feynman diagrams of the physically relevant QFTs could appear directly within the tensor categories we are using in our constructions.

This proposal is not out of harmony with the original ideas of Feynman [F], who seems to have believed that fundamental quantum processes were discrete, and to have been rather sceptical of continuum QFT as a framework. The point of view that perturbative QFT simply sidestepped unknown physics that appears at a very high energy has been a popular one for the subsequent development of perturbation theory.

We are proposing that a categorical state sum is the most natural candidate for the unknown high energy Physics.

We want to present some lines of argument to make this plausible.

In the first place, the Feynman rules have a very close formal similarity to the structure of a tensor category. Morphisms in a tensor category can be constructed using graphs, with objects on edges and tensor operators on vertices. Furthermore, duality in tensor categories gives an identification of the set of vertices with a certain number of lines in and the rest out with the vertices with the same number and type of lines, but the separation into in and out lines changed. For example, $Hom(A \otimes B, C \otimes D) = Hom(A \otimes C^*, B^* \otimes D)$. This is formally identical to Feynman's use of CPT invariance to identify vertices with edges going forward in time with similar edges going backwards.

The similarity between Feynmanology and categorical diagrams is not accidental. When one constructs a Lagrangian, one first determines what fields one needs to include, then arranges them as representations of the symmetries the theory is expected to have. Next, one looks for scalar combinations of the fields to include in the Lagrangian. These provide the possible vertices of the Feynmanology of the theory.

Note that representations of a group or algebra form a tensor category, and that by duality, scalars in the theory correspond to tensor operators. Thus, up to this point, the construction of tensor categories and Feynmanologies is identical.

Then there is the question of the Feynman integral, which gives us an evaluation of a Feynman diagram. In [B, F-K], it was pointed out that the evaluations of the closed diagrams in the categories of representations for Euclidean 4D GR (relativistic spin nets), were exactly Feynman integrals. In [BC2, R-R1], the same was shown for a regularized definition of an evaluation for closed diagrams in the category of unitary representations of the Lorentz algebra.

It might seem that the analogy between Feynmanology and tensor categories breaks down when we evaluate an open Feynman graph and get a number rather

than a morphism. This is not correct. The evaluation of a Feynman graph is a function of momenta for the incoming and outgoing particles. Passing to the momentum representation for the state spaces, we can regard these numbers as the matrix elements of a linear map. It is natural to think of this map as a morphism in a tensor category. The Ward or Slavnov-Taylor relations [t'H] amount to the statement that this map is an intertwiner for the gauge algebra, i.e. that it lives in the category of representations of the total symmetry of the system.

In the standard construction of a QFT, we would now select a few fields as the fundamental ones, find which vertices are renormalisable, and adjust the coupling constants in front of them to fit experiment. The picture of the vacuum in Feynmanological QFT is then in effect a subsummation of a categorical state sum, in which only special terms are counted, and with special numerical weights.

We now want to make a specific conjecture as to how Feynmanology as we know it arises.

CONJECTURE OF CATEGORICAL FEYNMANOLOGY: Let us postulate that just below the scale at which the hypergravity model goes topological, we have all the objects and morphisms of the category of representations of the physical symmetries of the standard model appearing in an effective state sum. This would correspond to a sum over Feynman diagrams, but allowing all possible particles and vertices. Let us then assume that the renormalization group acts on this theory, making all the unobserved types of particles become unstable in the low energy limit, and all the unrenormalizable vertices irrelevant (i.e. shrinking their coupling constants to near 0) and giving the remaining coupling constants their physical values.

The Feynman picture of the vacuum then appears as the low energy surviving remnant of the full categorical sum.

The remarkable thing about this conjecture is that it is really not so terribly radical. The idea that "all terms not forbidden by a symmetry appear in the effective Lagrangian" is commonplace in QFT. Particles are considered to be completely specified by their "quantum numbers" which just specify the representation of the physical symmetry which they form. The idea that the observed interactions are determined by the renormalization group is also standard.

It is not clear whether the conjecture above, which boils dow to the assumption that all possible fields appear at the fundamental scale and all vertices appear with equal coupling constants, has any physically testable consequences. It is arguable that it removes an esthetic problem from QFT. There has been the feeling that the Feynman revolution of the late 40s was disturbing in that it consisted of calculational tricks rather than a conceptual leap. If we really should think of summing over the entire category of representations of the total physical symmetry rather than merely the low energy limit, then Feynman in effect discovered the relationship between combinatorial topology and categor-

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ical algebra, which in some eyes is an elegant and fundamental departure from continuum Physics after all.

We are led to the idea that the Feynmanology of the standard model could appear directly as terms in our hypergravity TSS corresponding to representations of the world algebra, which, if our conjectures are true, would become important in an expansion around the constrained model in a low energy regime.

This suggests that although the strong force is nonperturbative in the infrared, physical calculations using perturbation theory for QCD could actually be done, since the onset of the topological theory would cut off the perturbation series. Terms with a large but finite number of loops would dominate the physics. Summing them would be equivalent to a categorical state sum on a triangulation with Planck scale spacings.

It is very tempting, in such a model, to take the ideas of Feynman more literally. Perhaps we might think of modifying our quasi-vacuum to include terms which fill the Dirac sea. It is also very natural to think of inserting Feynman vertices, with their invariance wrt time ordering into state sums representing general relativity, where different terms would correspond to different causal orderings.

This picture suggests that we attempt to generalize Feynmanology to the categorical setting. We could attempt to investigate such fundamental ideas of particle physics as renormalizability, anomalies, and current algebras in the greater generalization of a tensor category for example, using representations of a quantum group. Such a study would be inherently interesting, and might also help us to further the program of this paper.

5. Stringy Vacua

It was observed by Lee Smolin [S], that the natural perturbation theory around a classical solution in the Euclidean signature categorical state sums we are studying would resemble propagation of a string. This is because the natural basic perturbation to make to a labelled diagram representing a morphism in the category of representations of $U_q su(2)$ is to tensor by a constant spin around a loop. In fact, the binor calculus shows that any "spin net" as such diagrams are also known, is a superposition of such minimal perturbations.

It is not immediately obvious that this stringy picture would work as well in the Lorentzian signature, but that is a common enough problem in string theory.

Thus, if we want to study perturbation theory around a fixed term in a categorical state sum, we insert loops with spin labels, and then ask the probability that some other combination of basic perturbation loops appears later. This can be calculated by tracing the different possible paths for the loops, i.e. by summing over discrete world sheets for a discretized string.

Assuming for the moment that such a perturbation theory has some range of validity (a non-trivial assumption, but at least one which could be investigated),

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in what sort of space would it appear?

The CSSs we are using in this paper all begin by putting representations on the 2-simplices of a triangulated 4-manifold. Thus, the loops of representations we use as perturbations are occupying little circles of 2-simplices. If we think of the simplices as small, we can approximate these as circles in the bundle of bivectors over the spacetime 4-manifold.

Now the bundle of bivectors over a 4-manifold is a 10 dimensional manifold, fibered over the spacetime with a 6 dimensional fiber.

The natural suggestion is that in the limit of finer triangulations the perturbation theory would look like 10 dimensional string theory. Since the hypergravity multiplet contains both bosonic and fermionic terms with a natural functor connecting them, it is tempting to think it would reproduce superstring theory.

This further suggests that geometries in 10 dimensions could serve as quasivacua for the theory. One would decide what representation to assign to a 2-simplex by finding its area viewed as a surface in the bivector space endowed with a suitable metric.

We seem to be suggesting that the extra 6 dimensions for compactified models should be thought of not merely as small, but as infinitesimal, i.e. as bivectors associated to the tangent space.

This conjecture is principally motivated by the enormous work already existing on superstrings and string vacua. The only connections that are really solid at this point are the coincidence of dimensions, the hierarchy of bosonic and fermionic excitations, and the "stringiness" of the picture.

6 Generalizations. Membrane theory?

In broad outline, we are suggesting that the combination of a TSS and some interesting constrained subtheory might provide a mathematical setting for quantum physics. We have, in effect, begun to study the simplest possible such combination, corresponding to the lowest rank of noncompact quantum groups. It is intriguing that there is some indication that the standard model might emerge from the simplest example.

However, there will certainly be many other similar models which one could investigate. If the idea of obtaining the world algebra from the hypergravity multiplet didnt work, one could try bringing a gauge group directly into the theory by starting with a larger noncompact quantum group, for example. If nothing else, this would pose some very natural questions for representation theory.

Another possible generalization comes from the fact that the four dimensional TSS of [CKY1] from which we started is not the most general categorical construction possible. This is because it begins from a braided tensor category, whereas the most general 4D construction would use a spherical tensor 2-category [M1,2] or else a Hopf category [CF]. The construction we are using

is a special case, because a braided tensor category is a 2-category with one object.

The most general construction puts objects from the 2-category on edges of the triangulation, and 1-morphisms on faces. Since a braided tensor category is a 2-category with one object, the labelling of the edges becomes trivial and disappears.

In terms of the diagrammatic description, the edges of the diagram become boundaries of surfaces, which carry new labels, in the most general case.

At this point, I do not know of any physically significant tensor 2-categories. Interesting mathematical examples are constructed in [M2].

However, it is amusing to ask what would happen if we tried to find a geometric picture for a perturbation theory for a TSS model based on a general spherical tensor 2-category. The edges would correspond to bivectors as above, while the surfaces bounding them would live in the space of trivectors containing a bivector. The total space of such configurations is a 12 dimensional space. Thus, something like a membrane theory in 12d would be the natural choice for a geometric background. This is of course extremely speculative, but perhaps worth mentioning.

7. Conclusions. A Convergence of Ideas?

The direction we are mapping out is based on four ideas:

1. The fundamental geometry of spacetime is discrete, not continuous, hence should be constructed from a model on a triangulated manifold.

2.The model should be constructed from the structure of a tensor category, or if you prefer from Feynman integrals attached to the simplices.

3. The continuum limit should be fixed by the coming into play of a hierarchy of new fields, corresponding to more parts of the tensor category, which remove the ultraviolet problems by including an entire tensor category and hence becoming topological.

4. The largeness, flatness, and apparent complexity of the universe as we see it is due to an initial fluctuation, which caused it to take an apparent form more complex than the fundamental one.

Aside from the categorical formulation,these ideas are not new in theoretical physics. In particular, the picture of a multiplet of fields which comes into play only at high energies and softens out short distance behavior is very similar to supersymmetric theories.

What is new here is the application of unitary representations of noncompact groups, and the proposal to use the structure of the tensor category they form, and q-deformations of it directly to generate a physical model.

The model we are proposing is quite close to the spin foam picture [R-R2, Ba], which itself grew out of the loop variable program [R-S]. The most important difference between the two is the passage from a three dimensional to a four dimensional symmetry group, due to the adoption of a Lagrangian approach.

To the (perhaps somewhat partial) eye of the author, one of the remarkable features of the hypergravity model is the number of earlier ideas of twentieth century Physics to which it seems to give a home.

In addition to the spin foam/loop variables picture mentioned above, and to the possible connection with superstring theory, one could mention the old idea of Einstein [St] that the fundamental theory should be discrete and algebraic, Feynman's recasting of particle Physics as a discrete theory (as we can say now, of evaluations of words in a tensor category), Witten's idea of a "topological phase", and Connes derivation of the standard model.

Another intriguing connection is with the ideas of Majid [Maj]. Since the QLA is the quantum double of a quasitriangular Hopf algebra, it is a very canonical example of the crossed bialgebras he studies. Its role in the models in this paper is therefore reminiscent of the suggestions he makes relating them to quantum gravity. I do not know if his deeper ideas about duality will appear more directly in the development of this program.

Perhaps, even though each of these connections separately needs considerable work to make it more precise, the sheer number of natural connections to other streams of thought is worth noting.

We have made a large conceptual shift here from the mainstream of quantum Physics. We are replacing continuum geometry and fields by combinatorial geometry and categories of representations. The justification for any fundamental reformulation must be that it opens new possibilities for investigation.

It is much too soon to say that this program will solve the problems of theoretical physics. It is fair to say that it contains many directions to explore without obvious analogs in the continuum picture, and with suggestive connections to physical phenomenology. At a minimum, it can hardly fail to suggest interesting new questions in categorical algebra.

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